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Wall Heating Effects in Mixed Convection in Vertical Annulus with Variable Properties

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This paper concerns a numerical investigation of laminar, mixed convection in a concentric, circular annular duct. The inner and outer duct walls are maintained at uniform, equal dimensionless heat fluxes of $q^+_{iw} = q^+_{ow} = 5$ or $q^+_{iw} = q^+_{ow} = 1$. The thermophysical properties of the fluid vary with temperature. Results are presented for a Prandtl number of 0.7 for an annulus having a radius ratio of 0.25, using an implicit finite difference technique. It is shown that, at the lower wall heating rate, buoyancy can still enhance the local Nusselt number by as much as 30%.

Nomenclature

a, b, c = exponents in power law for specific heat, viscosity, and thermal conductivity, respectively
c_p = specific heat at constant pressure
c_p^+ = nondimensional specific heat, $c_p/c_{p,e}$
D_h = hydraulic diameter, $2(r_{ow} - r_{iw})$
f = friction factor, $2\tau_w/(\rho_m u_m^2)$
f_s = total friction factor, $(r^* \cdot f_{iw} + f_{ow})/(1 + r^*)$
Gr = modified Grashof number, $g(\rho/\mu)^2 D_h^3$
H = enthalpy
H^+ = nondimensional enthalpy, $(H-H_e)/(c_{p,e}T_e)$
h_m = mean heat transfer coefficient
k = thermal conductivity
k^+ = nondimensional thermal conductivity, k/k_e
Nu = local Nusselt number, $q_w^+/[k_b^+(T_w^+ - T_b)]$
P = pressure defect, $(p_e - p)/(\rho_e u_e^2)$
Pr = Prandtl number, $\mu_e c_{p,e}/k_e$
p = absolute pressure
p^+ = nondimensional pressure, p/p_e
Q = total heat absorbed by the fluid between channel entrance and any location x, $\int \rho u(H-H_e)2\pi r\, dr$
q''_w = heat flux at either wall
q^+ = nondimensional heat flux, $q''_w r_w/(k_e T_e)$
Re_e = Reynolds number, $\rho_e u_e D_h/\mu_e$
Re_m = Reynolds number, $\rho_m u_m D_h/\mu_m$
r = radial coordinate
r^+ = nondimensional radial coordinate, r/D_h
r^* = radius ratio, r_{iw}/r_{ow}
T = absolute temperature
T^+ = nondimensional temperature, T/T_e
u = axial velocity
u^+ = nondimensional axial velocity, u/u_e
v = radial velocity
v^+ = nondimensional radial velocity, $(v/u_e)Re_e Pr_e$
x = axial coordinate
x^+ = nondimensional axial coordinate, $(x/D_h)/(Re_e Pr_e)$
μ = viscosity
μ^+ = nondimensional viscosity, μ/μ_e
ρ = density
ρ^+ = nondimensional density, ρ/ρ_e
τ_w = wall shear stress

Subscripts
b = bulk quantity
e = for gas properties, reference value at $x = 0$; for nondimensional parameters, evaluated at $x = 0$
iw = inner wall
m = mean value (with respect to annulus cross section)
ow = outer wall

Introduction

RECENTLY, the subject of laminar convection in annular ducts has received some well-deserved attention in the published literature. Some of the studies that have appeared are concerned with buoyancy induced flow in annuli with end walls, so that the problem is that of convection in an enclosure; whereas other studies deal with natural convection or mixed convection in ducts with through flow.

Notable among the works on enclosure flow is that due to Hessami et al.,[1] who considered a horizontal duct of outer/inner radii ratio 11.4. They made measurements with air, glycerin, and mercury, and numerical computations were carried out for air and glycerin. Another study on enclosure flow was that by Littlefield and Desai.[2] Their numerical study was focused on the effect of boundary conditions. Existing papers that concern natural convection with through flow include that by Al-Arabi et al.[3] These authors conducted a numerical study based on boundary-layer equations for a vertical duct. Their approach was the same as that used for parallel plate ducts in natural convection[4,5] and mixed convection.[6,7] For mixed convection, recent publications are primarily concerned with fully developed flow.[8,9]

It is evident that natural convection in either closed- or open-ended vertical annular ducts has received some attention. In mixed convection, past studies have dealt with hydrodynamically and thermally fully developed flow, and no information is available on the combined influence of buoyancy and thermophysical property variations. We focus on

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this information gap in the present paper; that is, we study forced convection as it is influenced by the simultaneous effects of buoyancy and thermophysical property variations. In a companion paper,[10] the present approach has been validated by comparing with existing solutions for limiting situations. Aung and Moghadam[11] have applied the present method to examine the effect of temperature-dependent properties on the temperature field, the Nusselt number, and the friction factor. It is noted that, at high heating rates on the walls, the local Nusselt number is increased by as much as 50%. This raises the question of whether any significant influence of temperature-dependent properties exists at lower heating rates. The present paper provides results that address this issue.

Numerical Method

Details of the present numerical techniques are given in a separate paper,[10] and, therefore, only a brief summary will be given here. The problem at hand is hydrodynamically and thermally developing forced upward flow in a vertical, concentric, circular annular duct. The duct walls are heated at uniform heat fluxes. The flow is laminar, steady, and two dimensional and has a uniform velocity and temperature at the entrance to the duct. Viscous dissipation is neglected, but the thermophysical properties of the fluid are assumed to be temperature dependent. The density is given by the ideal gas equation of state. In order to facilitate ease of numerical solution, boundary-layer approximations are assumed to apply.

The dimensionless partial differential equations expressing the conservation of axial momentum, mass, and energy are given by Moghadam and Aung[10] and will not be repeated here. The axial momentum equation contains the parameter Gr/Re, which describes the buoyancy effect and is a specified quantity. The energy equation is expressed in terms of enthalpy in order to express the dependence of specific heat on temperature. In addition to these equations, the global mass balance equation is also needed, and the ideal gas equation of state is employed to give the density. Finally, properties are related to temperature through power-law relations. Thus, the following additional equations are relevant to the present problem:

$$\int_{r_{iw}^+}^{r_{ow}^+} \rho^+ u^+ r^+ \, dr^+ = \frac{1}{8}\frac{1 + r^*}{1 - r^*} \tag{1}$$

$$p^+ = \rho^+ T^+ \tag{2}$$

$$H^+ = \int_1^{T^+} c_p^+ \, dT^+ = \frac{1}{1 + a}[(T^+)^{1+a} - 1] \tag{3}$$

$$c_p^+ = (T^+)^a \qquad \mu^+ = (T^+)^b \qquad k^+ = (T^+)^c \tag{4}$$

The following commonly applied values are used for the exponents: $a = 0.095$, $b = 0.670$, and $c = 0.805$ in Eqs. (4). The boundary conditions used are the following:

At $x^+ = 0$, any r^+

$$u^+ = 1, \qquad v^+ = 0, \qquad H^+ = 0, \qquad P = 0$$

For $x^+ > 0$, on inner wall

$$u^+ = 0, \qquad v^+ = 0, \qquad \frac{\partial H^+}{\partial r^+} = \frac{c_p^+}{k^+} q_{iw}^+$$

For $x^+ > 0$, on outer wall

$$u^+ = 0, \qquad v^+ = 0, \qquad \frac{\partial H^+}{\partial r^+} = \frac{c_p^+}{k^+} q_{ow}^+$$

Values of wall heat fluxes considered are 5 and 1, in order to compare the influences of wall heating rate. Note that, owing to the way q^+ is defined, the physical heat transfer rate on the inner wall is actually four times higher than that on the outer wall. Also, the conditions $v^+ = 0$ at the boundaries may be derived from the global mass balance equation, Eq. (1), which implies no mass injection or suction at the walls.

A new set of variables may be defined as follows for graphical plotting purposes:

$$r_2^+ = \frac{r_1^+ - r^*}{1 - r^*}, \qquad r_1^+ = \frac{r}{r_{ow}}$$

The advantage of using r_2^+ is that it varies from 0 to 1.

This system of equations has been solved using an implicit finite difference technique that is based on earlier work by Bodoia and Osterle[4] and Aung et al.[5] The method has been shown by Hornbeck[12] to be unconditionally stable. Further details and validation of the method as applied to mixed convection in annular ducts are given by Moghadam and Aung.[10]

Results

In the present study, we have examined, for the first time, forced convection in a vertical annular duct with the combined effects of buoyancy and temperature-dependent thermophysical properties. Buoyancy effects are characterized by setting $1 \leq Gr/Re \leq 4 \times 10^3$, where $Gr/Re = 1$ corresponds to the forced convection limit.

It is of interest to gain an appreciation of the magnitude of the thermophysical property variations. Since pressure does not vary in the transverse direction, density is inversely proportional to temperature at a fixed axial location. The density profiles are, thus, the inverse of the temperature profiles there, as is indeed graphically demonstrated in this study. It is moreover seen that an increase in Gr/Re increases the density. Furthermore, it is found that it is entirely possible to have a nearly threefold variation in the density from the inner wall to the midplane of the duct. Increases in Gr/Re, however, decrease the thermal conductivity and dynamic viscosity. Figure 1 shows the radial distribution of the dimensionless thermal conductivity at different GR/Re. The trends across the duct are similar for the dynamic viscosity.

Figure 2 shows the velocity profile at $x^+ = 0.005$ for $q_{iw}^+ = q_{ow}^+ = 5$. In the pure forced convection limit, $Gr/Re = 1$, the profile shows the conventional shape with a maximum at the midplane. When buoyancy forces are significant, each of the profiles assumes a depression near the midplane of the duct. The depression is increased as Gr/Re becomes higher, and buoyancy forces adjacent to the heated wall draw more fluids toward the boundaries. The corresponding temperature

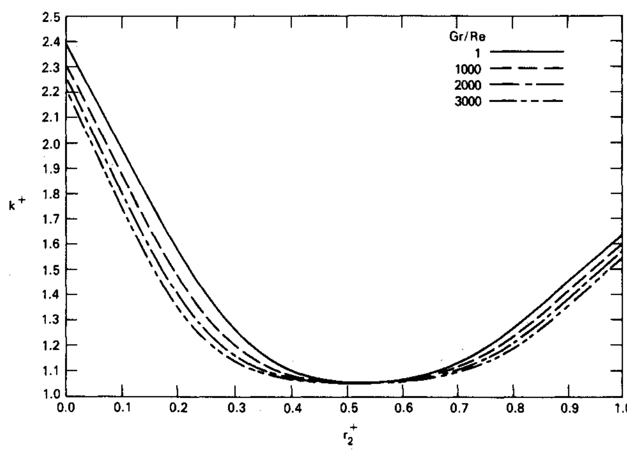

Fig. 1 Thermal conductivity profile at $x^+ = 0.005$ for super-imposed free and forced convection with variable properties: $q_{iw}^+ = q_{ow}^+ = 5$.

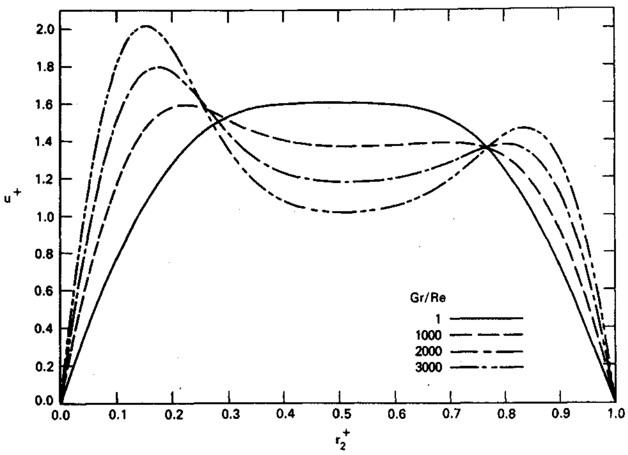

Fig. 2 Axial velocity profile at $x^+ = 0.005$ for $q^+_{iw} = q^+_{ow} = 5$.

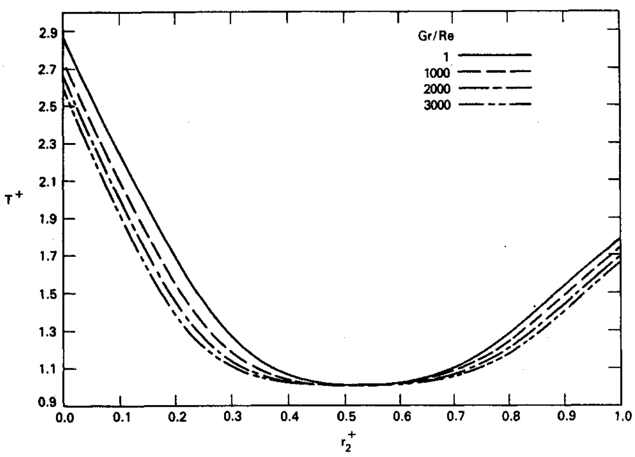

Fig. 3 Temperature profile at $x^+ = 0.005$ for $q^+_{iw} = q^+_{ow} = 5$.

distributions are given in Fig. 3. As indicated previously, the temperature distributions are simply the inverse of the density distributions. As Gr/Re changes from 1 to 3×10^3, the temperature on the hot wall decreases by about 10% and that on the cool wall decreases by about 7%.

A comparison of Fig. 2 with Fig. 3 shows that the effects of buoyancy are far more pronounced on the flowfield than on the thermal field. The implication is that mixed convection can have a major impact on the pumping power requirement, even though the influence on heat transfer rates is relatively slight. Results obtained in the present study indicate that this is, indeed, the case, as shown in the following.

One of the issues of concern in mixed convection flow in vertical ducts concerns the stability of the flow. Aung and Worku[6,7] have indicated that flow reversal can occur in gravity-aided mixed, laminar convection situations and have derived the criteria for flow reversal for parrallel duct channels that are sufficiently long. They showed that flow reversal is dependent on the heating conditions and the buoyancy parameter Gr/Re. Our present study has revealed no flow reversal in the range of parameters reported in the present paper; however, this may reflect a difference in the heating condition and/or geometry, and more study is needed.

Local Nusselt numbers on the outer wall are plotted in Fig. 4 for $q^+_{iw} = q^+_{ow} = 1$. At large x^+, the Nusselt numbers merge into the curve for pure forced convection. Further, at large axial distances, the local Nusselt number approaches an asymptotic, constant value. This value is the same for both walls. A similar conclusion may be drawn for the corresponding results for a higher heating rate of $q^+_{iw} = q^+_{ow} = 5$, as discussed by Aung and Moghadam.[11] In Ref. 10, it is shown that the inner wall Nusselt number increases by as much as

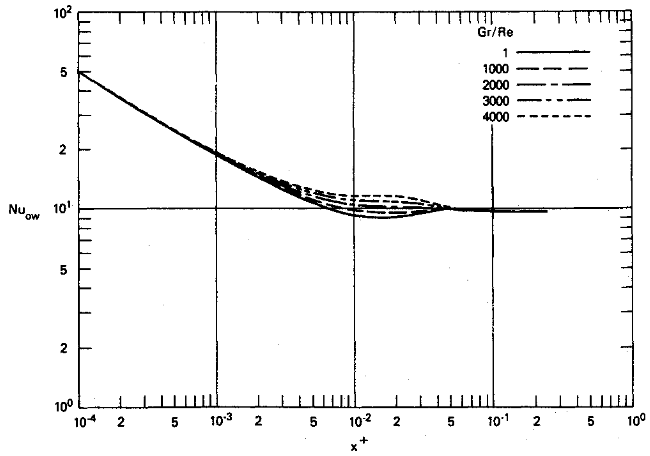

Fig. 4 Outer wall local Nusselt number for $q^+_{iw} = q^+_{ow} = 1$.

50% as a result of temperature-dependent fluid properties. It is seen from Fig. 4 that, at the lower heating of $q^+_{iw} = q^+_{ow} = 1$, the local Nusselt number on the outer wall still increases by a significant amount of up to 30%, the latter value being valid when $Gr/Re = 4 \times 10^4$.

The quantitative influence of buoyancy on the local Nusselt number is a function of the heating conditions. As can be expected, higher heating rates lead to larger influences due to buoyancy. In general, however, the present results show that the increase in the local Nusselt number due to buoyancy does not exceed about 50%.

Mean Nusselt numbers, based on the inner and outer wall log mean temperature differences (LMTD), are defined as follows:

$$Nu_{m,iw} = \frac{h_{m,iw} r_{iw}}{k_b}, \qquad Nu_{m,ow} = \frac{h_{m,ow} r_{ow}}{k_b}$$

where

$$h_{m,iw} = \frac{Q}{2\pi r_{iw} x . (\mathrm{LMTD})_{iw}}$$

$$h_{m,ow} = \frac{Q}{2\pi r_{ow} x . (\mathrm{LMTD})_{ow}}$$

$$(\mathrm{LMTD})_{iw} = \frac{T_b - T_e}{\ln[(T_{iw} - T_e)/(T_{iw} - T_b)]}$$

$$(\mathrm{LMTD})_{ow} = \frac{T_b - T_e}{\ln[(T_{ow} - T_e)/(T_{ow} - T_b)]}$$

In terms of dimensionless variables, the mean Nusselt numbers are given by

$$Nu_{m,iw} = \frac{\int_{r^+_{iw}}^{r^+_{ow}} \rho^+ u^+ H^+ r^+ \, dr^+}{k^+_b x^+ (\mathrm{LMTD})^+_{iw}}$$

$$Nu_{m,ow} = \frac{\int_{r^+_{iw}}^{r^+_{ow}} \rho^+ u^+ H^+ r^+ \, dr^+}{k^+_b x^+ (\mathrm{LMTD})^+_{ow}}$$

In these equations, $(\mathrm{LMTD})^+_{iw}$ and $(\mathrm{LMTD})^+_{ow}$ are appropriately nondimensionalized values of $(\mathrm{LMTD})_{iw}$ and $(\mathrm{LMTD})_{ow}$, respectively.

Mean Nusselt numbers for $q^+_{iw} = q^+_{ow} = 5$ are displayed in Fig. 5. Here, again, the effect of buoyancy is seen to be relatively small.

Pressure drop for the boundary condition $q^+_{iw} = q^+_{ow} = 5$ is shown in Fig. 6. The pressure drop increases dramatically

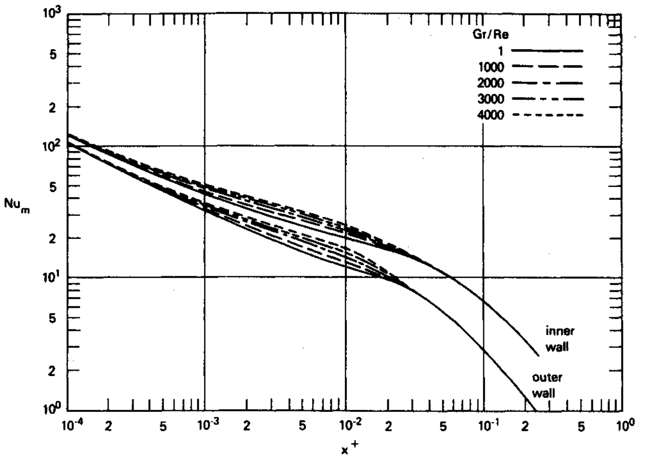

Fig. 5 Inner and outer wall mean Nusselt numbers for $q^+_{iw} = q^+_{ow} = 5$.

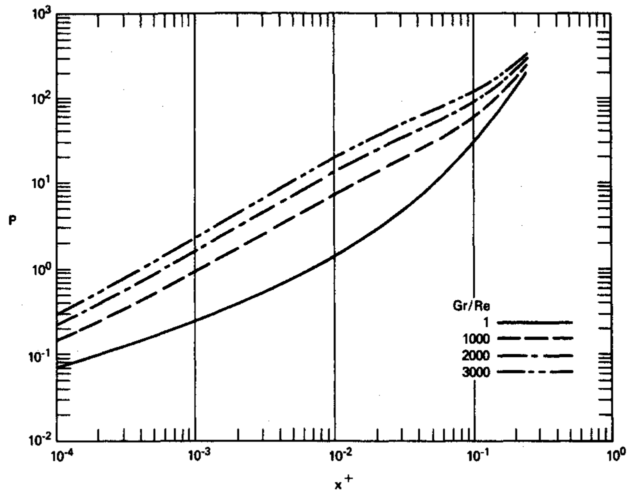

Fig. 6 Pressure defect distribution for $q^+_{iw} = q^+_{ow} = 5$.

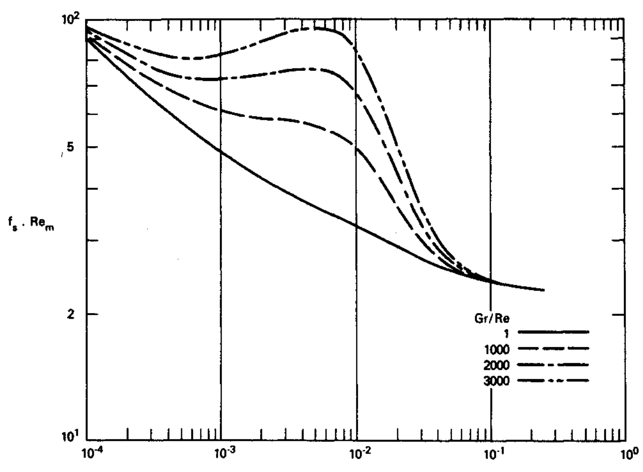

Fig. 7 Total friction factor distribution for $q^+_{iw} = q^+_{ow} = 5$.

with increasing Gr/Re. At $x^+ = 0.01$, the pressure drop corresponding to the largest Gr/Re is approximately 14 times that of the forced convection value. Therefore, one can conclude that mixed convection can have a major impact on pumping power requirement in thermal-fluids equipment. The impact, however, is felt only by channels of intermediate lengths because, for short and large axial distances, all curves tend to coincide with the curve for pure forced convection.

Total friction factors for $q^+_{iw} = q^+_{ow} = 5$ are shown in Fig. 7. It can be shown that

$$f_s . Re_m = \frac{2(r^* . \tau_{iw} + \tau_{ow})}{(1 + r^*)\mu^+_m u^+_m}$$

Buoyancy forces tend to increase the slope of the axial velocity profile near the walls leading to larger shear stresses. Higher values of Gr/Re result in lower values of mean viscosity and, therefore, give higher total friction factor. The overall trends are displayed in Fig. 7. Note that, again, the effects of buoyancy are limited to the intermediate axial distances.

Conclusions

In this paper, the combined influence of buoyancy and thermophysical property variations on laminar forced convection has been analyzed. The influence of wall heat flux level in which the dimensionless wall heat fluxes are equal has been examined. The thermophysical properties of the fluid are assumed to be temperature dependent. It is shown that properties change by a factor of more than 2 across the vertical annular duct. The effect of buoyancy on the axial distribution of the local Nusselt number is significant for both the inner and outer walls. The local Nusselt number is shown to increase due to property variation, but the increase depends on the wall heat flux. At the higher dimensionless heat flux of 5, the increase is approximately 50%, whereas the increase is about 30% for dimensionless wall heat flux of 1. The effects of buoyancy on the pressure drop and friction factor are very significant, but the effects depend on the channel length. For short and long channels, the pressure drop and friction factor coincide with the forced convection values.

Acknowledgments

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